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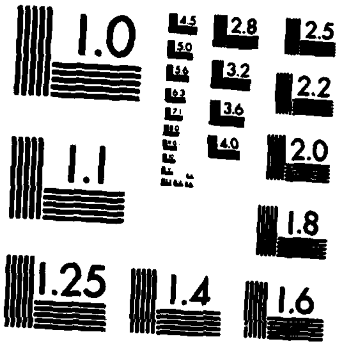


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Final Scientific Report

# Nonlinear Partial Differential Equations and Related Problems of Padé Approximations

by D.V. Chudnovsky and G.V. Chudnovsky



The focus of ~~our~~ the authors' work is the understanding and possibly explicit mathematical solution of the physically realistic classical and quantum models of field theories in dimensions one to four. ~~We~~ They aim at the description of hidden symmetries and analytic structures of the solutions of various physical systems in dimensions one-four described by systems of nonlinear p.d.e. in cases when a system is suspected of being completely integrable, or at the investigation of obstructions to complete integrability.

We continued and expanded our earlier studies of the structure of infinite-dimensional Lie algebras generated by Bäcklund transformations and the geometric interpretation of the structure of S-matrices, applied to the extension of the Bethe Ansatz determining the solutions of completely integrable quantum systems to three- and four-dimensional systems. Very little is currently known in these cases, and there arise new multidimensional generalizations of factorization eqautions of S-matrices that have to be solved. A totally new area for mathematical studies has been opened by recent progress in the physics of

superstrings, where we hope to apply our methods of multi-particle and multi-string interactions, and where the use of methods from algebraic topology and theta functions (some of which were inroduced by us in lower-dimensional cases earlier) seems very promising.

Our work includes the theoretical and computer study of Padé approximations and their applications, particularly to differential equations, differential algebra and number theory. Methods of Padé approximations proved to be very powerful in the study of completely integrable systems (particularly in the description of Bäcklund transformations). These methods allowed us to achieve a major progress in the study of the analytic and arithmetic properties of solutions of differential equations. Extending our pevious work (e.g. the solution to Kolchin's problem and the proof of the "almost perfecness" of Padé approximations to algebraic functions), we have solved in many cases the Grothendieck conjecture. According to it, the monodromy structure of a linear differential equation (more precisely, a Lie algebra of a Galois group of an equation) can be determined from modp properties of this equation for (a finite set of) primes p. This led to a development and implementation of new algorithms that determine the algebraicity of solutions of differential equations, and reduction of Abelian integrals. Methods of Padé approximations in combination with formal groups made it possible for us to solve effectively Tate's conjecture in many cases. The work on formal groups and Padé approximations leads to many number-theoretic applications, and to some more practical applications to new classes of computer algorithms for primality testing.

Our work greately benefits from the use of various computer

algebra systems. We are working on the design, optimization and implementation of specialized computer algebra programs, particularly for very fast power series manipulations, Padé and continued fraction approximations, and fast numeric and symbolic solutions of differential equations.

During the period of the Grant (since 1984) we had prepared, with the support of Air Force, and published 13 research papers, 2 papers were prepared for publications. We were authors or editors of 4 books or proceedings of seminars.

In 1984 we were among organizers and speakers of a conference "Computer Algebra as a Tool for Research in Mathematics and Physics", held in New York (D.V. was a co-chairman of the Conference). There were more than 500 participants at the Conference. A new conference on "Computers and Mathematics" will be held in July - August 1986 in Stanford.

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Recent publications of D.V. Chudnovsky and G.V. Chudnovsky for the Project "Nonlinear Partial Differential Equations and Related Problems of Padé Approximations", supported by the U.S. Air Force.


1. D.V. Chudnovsky, Topological and algebraic structures of linear problems associated with completely integrable systems, in Proceedings of XI International Conference in Group Theoretical Methods in Physics, Istanbul, 1982.

   Lecture Notes Physics, v. 180, 1983, 65-90.

2. D.V. Chudnovsky, Bäcklund transformations and geometric and complex-analytic background for construction of completely integrable lattice systems,

   In Problems of Mathematics and Physics, Festschrift in honor of F. Gursey, Gordon and Breach, 1984, 46pp (in press).

3. G.V. Chudnovsky, Bäcklund transformations and deformations of linear differential equations with applications to diophantine approximations, in Problems of Mathematical Physics.

   Festschrift in honor of F. Gursey, Gordon and Breach, 1984, 30 pp. (in press).

4. G.V. Chudnovsky, Number theoretical applications of polynomials with rational coefficients defined by extremality conditions, in Arithmetic and Geometry.

   Progress in Mathematics, v. 35, Birkhauser, Boston, 1983, pp. 67-107.

5. D.V. Chudnovsky, G.V. Chudnovsky, Algebraic reductions of scalar three-dimensional systems,

   Phys. Lett. A, 93A(1983), 371-374.

6. D.V. Chudnovsky, G.V. Chudnovsky, Laws of composition of Bäcklund transformations and the universal form of completely integrable systems in dimensions two and three.

7. G.V. Chudnovsky, On the method of Thue-Siegel.

   Ann. of Math. v. 117 (1983), 325-382.

8. D.V. Chudnovsky, G.V. Chudnovsky, Rational approximations to solutions of linear differential equations.

   Proc. Nat'l. Acad. Sci. U.S.A., v. 80 (1983), 5158-5162.

9. D.V. Chudnovsky, G.V. Chudnovsky, M. Tabor, Painlevé property and multicomponent isospectral deformation equations.

Phys. Lett. A., 97A, 1983, 268-274.

10. G.V. Chudnovsky, The Thue-Siegel-Roth Theorem for values of algebraic functions.

Proc. Japan Academy, v.59, 1983, 281-284.

11. G.V. Chudnovsky, Rational approximations to linear forms of exponential and binomials.

Proc. Nat'l. Acad. Sci. U.S.A., v.80, 1983, 3183-3141.

12. D.V. Chudnovsky, G.V. Chudnovsky, Padé and rational approximations to system of functions and their arithmetic applications.

Lecture Notes in Mathematics, Springer, N.Y., v.1052, 1984, 37-84.

13. D.V. Chudnovsky, G.V. Chudnovsky, Padé approximations to solutions of linear differential equations and applications to diophantine analysis.

Lecture Notes in Mathematics, Springer, N.Y.,v.1052, 1984, 85-167.

14. D.V. Chudnovsky, G.V. Chudnovsky, Some remarks on theta functions and S-matrices.

In Classical and Quantum Models and Arithmetic Problems, M. Dekker, v.92, 1984, 100pp.

15. D.V. Chudnovsky, G.V. Chudnovsky, Note on Eisenstein's system of differential equations.

In Classical and Quantum Models and Arithmetic Problems, M. Dekker, v.92, 1984, 17pp.

16. D.V. Chudnovsky, G.V. Chudnovsky, Recurrences, Padé Approximations and their applications.

In Classical and Quantum Models and Arithmetic Problems, M. Dekker, v.92, 1984, 25pp.

17. G.V. Chudnovsky, On some applications of diophantine approximations.

Proc. Nat'l. Acad. Sci. U.S.A.,v.81, 1984, 1926-1930.

18. D.V. Chudnovsky, G.V. Chudnovsky, The Wronskian Formalism for Linear Differential Equations and Padé Approximations.

Advances in Mathematics, v.52, 1984, 111-138.

19. G.V. Chudnovsky, On applications of diophantine approximations.
Proc. Nat'l. Acad. Sci. U.S.A., v. 81 (1984), 7261-7265.

20. D.V. Chudnovsky, G.V. Chudnovsky, The Grothendieck conjecture and Padé approximations.
Proc. Japan Acad., v. 61, Ser. A (1985), 87-91.

21. D.V. Chudnovsky, G.V. Chudnovsky, Padé approximations and diophantine geometry.
Proc. Nat'l. Acad. Sci. U.S.A., v. 82 (1985), 2212-2216.

22. D.V. Chudnovsky, G.V. Chudnovsky, Applications of Padé Approximations to Diophantine Inequalities in Values of G-Functions.
Lecture Notes in Mathematics, Springer, N.Y., v. 1135 (1985), 44pp.

23. D.V. Chudnovsky, G.V. Chudnovsky, Applications of Padé Approximations to the Grothendieck Conjecture on Linear Differential Equations.
ibid, 54pp.

24. D.V. Chudnovsky, G.V. Chudnovsky, Sequences of numbers generated by addition in formal groups and new primality and factorization tests, IBM Research Report, RC 11262, 7/12/85.

25. D.V. Chudnovsky, G.V. Chudnovsky, A random walk in higher arithmetic, Advances in Appl. Math., 1986 (to appear).

26. D.V. Chudnovsky, G.V. Chudnovsky, Elliptic Modular Functions and Elliptic Genera, Topolgy (submitted).

## BOOKS

1. Number Theory, Proceedings of New York Seminar, 1981-1982, Lecture Notes in Mathematics, v. 1052, Springer-Verlag, NY, 1984, 309 pp. (edited by D.V. Chudnovsky, G.V. Chudnovsky, H. Cohn and M. Nathanson).

2. Classical and Quantum Models and Arithmetic Problems, Lecture Notes in Pure and Applied Mathematics, M. Dekker, NY, 1984, 461 pp. (edited by D.V. Chudnovsky, G.V. Chudnovsky).

3. Contributions to the Theory of Transcendental Numbers, by G.V. Chudnovsky, mathematical Surveys and Monographs, v. 19, American Mathematical Society, Providence, Rhode Island, 1984, XI, 450 pp.

4. Number Theory, Proceedings of New York Seminar, 1983-1984, Lecture Notes in Mathematics, v. 1135, Springer-Verlag, NY, 1985, 283 pp. (edited by D.V. Chudnovsky, G.V. Chudnovsky, H. Cohn and M. Nathanson).

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